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Economic Analysis of a Computer System with Software Up-Gradation and Priority to Hardware Repair over Hardware Replacement Subject to Maximum Operation and Repair Times

Ashish Kumar, Monika S. Barak & S.C.Malik

Abstract— The main objective of this paper is to make economic analysis of a computer system of two identical units- one is operative and other is kept as cold standby. In each unit h/w and s/w fails independently directly from normal mode. There is a single server who visits the system immediately to conduct preventive maintenance, h/w repair, h/w replacement and s/w up-gradation. The preventive maintenance of the system is carried out after a maximum operation time. If repair of the h/w is not possible by the server up to a pre-specific time (called Maximum Repair Time), it is replaced by new one with some replacement time. However, s/w up-gradation is made whenever s/w fails to meet out the desired function properly. Priority to h/w repair is given only over h/w replacement. The failure time of h/w and s/w follow negative exponential distribution while the distributions of preventive maintenance, h/w repair, h/w replacement and s/w up-gradation times are taken as arbitrary with different probability density functions. Graphs are drawn for a particular case to show the behaviour of MTSF, availability and profit function with preventive maintenance rate and fixed values of other parameters.

Index Terms— Computer System, H/W and S/W Failure, Maximum Operation and Repair Time, Preventive Maintenance and Economic Measures..

1 Introduction

THE increasing dependency of today's society on computer systems makes the field of reliability and performance evaluation of computers highly important. Generally, reliability of a computer system depends on the performance of its h/w and s/w components. H/w and s/w works together in most of the computing systems to provide computerized functionality. When the requirements and dependencies on computer systems increase, the possibility of their failure also increases. Generally, there are two types of failures in a computer system- h/w failure and s/w failure. The impact of these failures ranges from inconvenience to economic damages to loss of life. Therefore, it is important to operate such systems with high reliability. A few researcher including Friedman and Tran (1992) and Welke et al. (1995) tried to establish a combined reliability model for the whole system including both H/W and S/W. Redundancy is one of the best method to improve the reliability of any operating systems. Therefore, in recent years, stochastic models of two-unit cold standby computer systems having independent h/w and s/w failures have been suggested by some researchers including Malik and Anand (2010) and Malik and Kumar (2011). On the other hand, preventive maintenance can slow the adulterate process of a computer system and restore the system as new. Thus, the method of preventive maintenance can be adopted to improve the reliability and profit of system.

The concept of preventive maintenance has been used by Malik and Nandal (2010) while analyzing a redundant system with maximum operation time. Also, sometimes, it becomes necessary to give priority in repair to one unit over repair activities of other unit not only to reduce the down time but also to minimize the operating cost. Singh and Agrafiotis (1995) analyzed stochastically a two-unit cold standby system subject to maximum operation and repair time. Furthermore, reliability and availability of a system can be increased by making replacement of the failed component by new one in case repair time is too long. Recently, Malik and Kumar (2012) investigate reliability models for a computer system with preventive maintenance and repair subject to maximum operation and repair times.

Keeping in mind the above facts, here a stochastic model for a computer system of two identical units - one is operative and other is kept as spare in cold standby is developed. In each unit h/w and s/w fails independently. There is a single server who visits the system immediately to do preventive maintenance, h/w repair, h/w replacement and s/w up-gradation. The preventive maintenance of the system is carried out after a maximum operation time. If the server is unable to repair the h/w up to a pre-specific time (called Maximum Repair Time), it is replaced by new one with some replacement time. However, s/w is up-graded upon its failure. Priority to h/w repair is given only over h/w replacement. The expressions various measures of system effectiveness such as mean time to system failure, availability , busy period

Ashish Kumar & Monika S. Barak working at Department of Mathematics & Statistics, Manipal University Jaipur, Jaipur-303007, India

Dr. S.C. Malik working as Professor at Department of Statistics, M. D. University, Rohtak-124001, India

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of the server due to preventive maintenance, busy period of the server due to h/w repair, busy period of the server due to hardware replacement , busy period of the server due to software up-gradation, expected number of software up-gradations , expected number of hardware replacement and expected number of visits of the server are derived by using semi-Markov process and regenerative point technique. All random variables are statistically independent and uncorrelated. Switch devices are perfect. The graphical study of the results for a particular case has also been made to highlight the importance of the results.

2 Notations

E	:	The set of regenerative states
No	:	The unit is operative and in normal mode
Cs	:	The unit is cold standby
a/b	:	Probability that the system has hardware / software failure
λ_1/λ_2	:	Constant hardware / software failure rate
α_0	:	Maximum Operation Time
β_0	:	Maximum Repair Time.
Pm/PM	:	The unit is under preventive Maintenance/ under preventive maintenance continuously from previous state
WPm/WPM	:	The unit is waiting for preventive Maintenance/ waiting for preventive maintenance from previous state
HFur/HFUR	:	The unit is failed due to hardware and is under repair / under repair continuously from previous state
HFurp/HFURP	:	The unit is failed due to hardware and is under replacement /under replace ment continuously from previous state
HFwr / HFWR	:	The unit is failed due to hardware and is waiting for repair/waiting for repair continuously from previous state
SFurp/SFURP	:	The unit is failed due to the software and is under up-gradation/under up-gradation continuously from previous state
SFwrp/SFWRP	:	The unit is failed due to the softwar and is waiting for Up-gradation / wait ing for up-gradation continuously from previous state
h(t) / H(t)	:	pdf / cdf of s/w up-gradation time
g(t) / G(t)	:	pdf / cdf of repair time of the hardware
m(t)/ M(t)	:	pdf / cdf of replacement time of the hardware
f(t) / F(t)	:	pdf / cdf of the time for PM of the unit
$q_{ij}(t)$/ $Q_{ij}(t)$	:	pdf / cdf of passage time from regenera tive state i to a regenerative state j or to a failed state j without visiting any othe r regenerative state in (0, t]
pdf / cdf	:	Probability density function/ Cumulat ive density function
$q_{ij.kr}(t)/Q_{ij.kr}(t)$	:	pdf/cdf of direct transition time from regenerative state i to a regenerative state j or to a failed state j visiting state k, r once in (0, t]
$\mu_i(t)$		Probability that the system up initially in state $S_i \in E$ is up at time t without visiting to any regenerative state
$W_i(t)$		Probability that the server is busy in the state S_i upto time 't'without making any transition to any other regenerative state or returning to the same state via one or more non-regenerative states.
m_{ij}		Contribution to mean sojourn time (μ_i) in state S_i when system transit directly to state S_j so that $\mu_i = \sum_j m_{ij}$ and $m_{ij} = \int t dQ_{ij}(t) = -q_{ij}^{*\prime}(0)$

3 Transition Probabilities and Mean Sojourn Times

Simple probabilistic considerations yield the following expressions for the non-zero elements

$$p_{ij} = Q_{ij}(\infty) = \int_0^\infty q_{ij}(t)dt \quad \text{as} \tag{1}$$

$p_{01} = \dfrac{\alpha_0}{a\lambda_1 + b\lambda_2 + \alpha_0}$, $p_{02} = \dfrac{a\lambda_1}{a\lambda_1 + b\lambda_2 + \alpha_0}$,

$p_{03} = \dfrac{b\lambda_2}{a\lambda_1 + b\lambda_2 + \alpha_0}$, $p_{10} = f^*(a\lambda_1 + b\lambda_2 + \alpha_0)$,

$p_{16} = \dfrac{a\lambda_1}{a\lambda_1 + b\lambda_2 + \alpha_0}[1 - f^*(a\lambda_1 + b\lambda_2 + \alpha_0)] = p_{12.6}$,

$p_{18} = \dfrac{b\lambda_2}{a\lambda_1 + b\lambda_2 + \alpha_0}[1 - f^*(a\lambda_1 + b\lambda_2 + \alpha_0)] = p_{13.8,}$

$p_{1.13} = \dfrac{\alpha_0}{a\lambda_1 + b\lambda_2 + \alpha_0}[1 - f^*(a\lambda_1 + b\lambda_2 + \alpha_0)] = p_{11.13}$,

$p_{20} = g^*(a\lambda_1 + b\lambda_2 + \alpha_0 + \beta_0)$, $p_{24} = \dfrac{\beta_0}{a\lambda_1 + b\lambda_2 + \alpha_0 + \beta_0}[1 - g^*($ $a\lambda_1 + b\lambda_2 + \alpha_0 + \beta_0)]$, $p_{25} = \dfrac{\alpha_0}{a\lambda_1 + b\lambda_2 + \alpha_0 + \beta_0}[1 - g^*($

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$a\lambda_1+b\lambda_2+\alpha_0+\beta_0)]p_{2.11} = \dfrac{b\lambda_2}{a\lambda_1+b\lambda_2+\alpha_0+\beta_0}[1-g^*(a\lambda_1+b\lambda_2+\alpha_0+\beta_0)]$, $p_{2.12} = \dfrac{a\lambda_1}{a\lambda_1+b\lambda_2+\alpha_0+\beta_0}[1-g^*(a\lambda_1+b\lambda_2+\alpha_0+\beta_0)]$, $p_{30} = h^*(a\lambda_1+b\lambda_2+\alpha_0)$,

$p_{37} = \dfrac{a\lambda_1}{a\lambda_1+b\lambda_2+\alpha_0}[1-h^*(a\lambda_1+b\lambda_2+\alpha_0)] = p_{32.7}$,

$p_{39} = \dfrac{\alpha_0}{a\lambda_1+b\lambda_2+\alpha_0}[1-h^*(a\lambda_1+b\lambda_2+\alpha_0)] = p_{3,1.9}$,

$p_{40} = m^*(a\lambda_1+b\lambda_2+\alpha_0)$, $p_{3,10} = \dfrac{b\lambda_2}{a\lambda_1+b\lambda_2+\alpha_0}[1-h^*(a\lambda_1+b\lambda_2+\alpha_0)] = p_{33.10}$, $p_{51} = g^*(\beta_0)$, $p_{5,16} = 1-g^*(\beta_0)$,

$p_{4.17} = \dfrac{\alpha_0}{a\lambda_1+b\lambda_2+\alpha_0}[1-m^*(a\lambda_1+b\lambda_2+\alpha_0)] = p_{4,1.17}$,

$p_{62} = f^*(0)$, $p_{72} = h^*(0)$, $p_{83} = f^*(0)$, $p_{93} = f^*(0)$, $p_{10.3} = h^*(0)$, $p_{11.3} = g^*(\beta_0)$, $p_{11.14} = 1-g^*(\beta_0)$, $p_{4,18} = \dfrac{b\lambda_2}{a\lambda_1+b\lambda_2+\alpha_0}[1-m^*(a\lambda_1+b\lambda_2+\alpha_0)] = p_{43.18}$, $p_{12.2} = g^*(\beta_0)$, $p_{12.15} = 1-g^*(\beta_0)$, $p_{13.1} = f^*(0)$, $p_{14.3} = m^*(0)$, $p_{15.2} = m^*(0)$, $p_{16.1} = m^*(0)$,

$p_{4.19} = \dfrac{a\lambda_1}{a\lambda_1+b\lambda_2+\alpha_0}[1-m^*(a\lambda_1+b\lambda_2+\alpha_0)]$,

$p_{17.1} = m^*(0)$, $p_{18.3} = m^*(0)$, $p_{19.4} = g^*(0)$,

$p_{21.5} = \dfrac{\alpha_0}{a\lambda_1+b\lambda_2+\alpha_0+\beta_0}[1-g^*(a\lambda_1+b\lambda_2+\alpha_0+\beta_0)]\,g^*(\beta_0)$,

$p_{21.16,5} = \dfrac{\alpha_0}{a\lambda_1+b\lambda_2+\alpha_0+\beta_0}[1-g^*(a\lambda_1+b\lambda_2+\alpha_0+\beta_0)][1-g^*(\beta_0)]$, $p_{23.11} = \dfrac{b\lambda_2}{a\lambda_1+b\lambda_2+\alpha_0+\beta_0}[1-g^*(a\lambda_1+b\lambda_2+\alpha_0+\beta_0)][g^*(\beta_0)]$, $p_{23.11,14} = \dfrac{b\lambda_2}{a\lambda_1+b\lambda_2+\alpha_0+\beta_0}[1-g^*(a\lambda_1+b\lambda_2+\alpha_0+\beta_0)][1-g^*(\beta_0)]$, $p_{22.12} = \dfrac{a\lambda_1}{a\lambda_1+b\lambda_2+\alpha_0+\beta_0}[1-g^*(a\lambda_1+b\lambda_2+\alpha_0+\beta_0)]\,g^*(\beta_0)$, $p_{22.12,15} = \dfrac{a\lambda_1}{a\lambda_1+b\lambda_2+\alpha_0+\beta_0}[1-g^*(a\lambda_1+b\lambda_2+\alpha_0+\beta_0)][1-g^*(\beta_0)]$ (2)

It can be easily verified that $p_{01}+p_{02}+p_{03} = p_{10}+p_{16}+p_{18}+p_{1.13} = p_{20}+p_{24}+p_{25}+p_{2,11}+p_{2.12} = p_{30}+p_{37}+p_{39}+p_{3,10} = p_{40}+p_{4.17}+p_{4.18}+p_{4.19} = p_{5.1}+p_{5.16} = p_{62} = p_{72} = p_{83} = p_{91} = p_{10.3} = p_{11.3}+p_{11.14} = p_{12.2}+p_{12.15} = p_{13.1} = p_{14.1} = p_{15.2} = p_{16.1} = p_{17.1} = p_{18.3} = p_{19.4} = p_{10}+p_{12.6}+p_{11.13}+p_{13.8} = p_{20}+p_{24}+p_{21.5}+p_{21,16.5}+p_{23,11}+p_{23.11,14}+p_{22,12}+p_{22.12,15} = p_{30}+p_{31.9}+p_{32.7}+p_{33.10} = p_{40}+p_{41.17}+p_{42.19}+p_{43.18} = 1$ (3)

The mean sojourn times (μ_i) is the state S_i are

$\mu_0 = \dfrac{1}{a\lambda_1+b\lambda_2+\alpha_0}$, $\mu_1 = \dfrac{1}{a\lambda_1+b\lambda_2+\alpha_0+\alpha}$, $\mu_2 = \dfrac{1}{a\lambda_1+b\lambda_2+\alpha_0+\theta+\beta_0}$, $\mu_3 = \dfrac{1}{a\lambda_1+b\lambda_2+\alpha_0+\beta}$, $\mu_4 = \dfrac{1}{a\lambda_1+b\lambda_2+\alpha_0+\gamma}$, (4)

The states S0, S1, S2, S3 and S4 are regenerative states while S5, S6, S7, S8, S9, S10, S11, S12, S13, S14, S15, S16, S17, S18 and S19 are non-regenerative states. Thus E = {S0, S1, S2, S3, S4}.The possible transition between states along with transition rates for the model is shown in figure 1.

4 RELIABILITY MEASURES

4.1 RELIABILITY AND MEAN TIME TO SYYSTEM FAILURE

Let $\phi_i(t)$ be the c.d.f of first passage time from the regenerative state i to a failed state. Regarding the failed state as absorbing state, we have the following recursive relation for $\phi_i(t)$:

$$\phi_i(t) = \sum_j Q_{i,j}(t) ® \phi_j(t) + \sum_k Q_{i,k}(t) \qquad (5)$$

Where *j* is an un-failed regenerative state to which the given regenerative state *i* can transit and *k* is a failed state to which the state *i* can transit directly.

Taking LST of above relation (5) and solving for $\tilde{\phi}_0(s)$.

We have

$$R^*(s) = \frac{1-\tilde{\phi}_0(s)}{s} \qquad (6)$$

The reliability of the system model can be obtained by taking Laplace inverse transform of (6).

The mean time to system failure (MTSF) is given by

$$\text{MTSF} = \lim_{s\to o}\frac{1-\tilde{\phi}_0(s)}{s} = \frac{N_1}{D_1} \quad \text{where} \qquad (7)$$

$N_1 = \mu_0 + p_{01}\mu_1 + p_{02}\mu_2 + p_{03}\mu_3 + p_{24}p_{02}\mu_4$

and $D_1 = 1 - p_{01}p_{10} - p_{02}p_{20} - p_{03}p_{30} - p_{02}p_{24}p_{40}$

4.2 AVAILABILITY

Let $A_i(t)$ be the probability that the system is in up-state at instant 't' given that the system entered regenerative state i at t = 0. The recursive relations for $A_i(t)$ are given as

$$A_i(t) = M_i(t) + \sum_j q_{i,j}^{(n)}(t) © A_j(t) \qquad (8)$$

Where *j* is any successive regenerative state to which the regenerative state *i* can transit through n transitions. $M_i(t)$ is the probability that the system is up initially in state $S_i \in E$ is up at time t without visiting to any other regenerative state, we have

$$M_0(t) = e^{-(a\lambda_1 + b\lambda_2 + \alpha_0)t}, M_1(t) = e^{-(a\lambda_1 + b\lambda_2 + \alpha_0)t}\overline{F(t)},$$
$$M_2(t) = e^{-(a\lambda_1 + b\lambda_2 + \alpha_0 + \beta_0)t}\overline{G(t)} \; M_3(t) = e^{-(a\lambda_1 + b\lambda_2 + \alpha_0)t}\overline{H(t)},$$
$$M_4(t) = e^{-(a\lambda_1 + b\lambda_2 + \alpha_0)t}\overline{M(t)}$$

Taking LT of above relations (8) and solving for $A_0^*(s)$, the steady state availability is given by

$$A_0(\infty) = \lim_{s\to 0} sA_0^*(s) = \frac{N_2}{D_2}, \text{ where}$$

$N_2 = (-p_{24})\{\mu_0 \, [(1- p_{11.13}) \, p_{43.18} \, p_{32.7} + p_{32.7}p_{41.17}p_{13.8} + p_{12.6}\{ p_{43.18}p_{31.9} + p_{41.17}(1- p_{33.10})\} - \mu_1 \, [-p_{01} \, p_{43.18} \, p_{32.7} + p_{03} \, p_{41.17} \, p_{32.7} - + p_{02}\{ p_{43.18}p_{31.9} + (1- p_{33.10})p_{41.17}\}] + \mu_3 \, [-p_{01} \, p_{43.18} \, p_{12.6} + p_{03} \, p_{41.17} \, p_{12.6} + p_{02}\{ p_{43.18} \, (1-p_{11.13}) + p_{41.17}p_{13.8} \}] - \mu_4 \, [p_{01} \, \{(1- p_{33.10})p_{12.6} + p_{32.7} \, p_{13.8} \} + p_{02}\{(1- p_{11.13}) \, (1- p_{33.10}) - p_{13.8}p_{31.9}\} + p_{03}\{(1- p_{11.13}) \, p_{32.7} + p_{31.9}p_{12.6} \}]\} + (1- p_{4.19}p_{19.4})\{ \mu_0 \, [(1- p_{11.13}) \, (1- p_{33.10}) \, (1- p_{22.12} - p_{22.12.15}) - (p_{23.11} + p_{23,11.14})p_{32.7}] + p_{12.6}\{ -(1- p_{33.10}) \, (p_{21.5} + p_{21.5,16}) - (p_{23.11} + p_{23,11.14})p_{31.9}\} - p_{13.8}\{ p_{32.7} \, (p_{21.5} + p_{21.5,16}) + (1- p_{22.12} - p_{22.12.15})p_{31.9}\}] + \mu_1[p_{01} \, [(1- p_{33.10}) \, (1- p_{22.12} - p_{22.12.15}) - (p_{23.11} + p_{23,11.14})p_{32.7}] + p_{02}\{ (1- p_{33.10}) \, (p_{21.5} + p_{21.5,16}) + (p_{23.11} + p_{23,11.14})p_{31.9}\} + p_{03}\{ p_{32.7} \, (p_{21.5} + p_{21.5,16}) + (1- p_{22.12} - p_{22.12.15})p_{31.9}\}] + \mu_2 \, [p_{01} \, \{(1- p_{33.10}) \, p_{12.6} + p_{13.8}p_{32.7}\} + p_{02}\{(1- p_{11.13}) \, (1- p_{33.10}) - p_{13.8}p_{31.9}\} + p_{03}\{(1- p_{11.13}) \, p_{32.7} + p_{31.9}p_{12.6} \} + \mu_3 \, [p_{01} \, [p_{13.8}(1- p_{22.12} - p_{22.12.15}) + (p_{23.11} + p_{23,11.14})p_{12.6}] + p_{02}\{ (p_{21.5} + p_{21.5,16}) \, p_{13.8} + (1- p_{11.13}) \, (p_{23.11} + p_{23,11.14}) \} + p_{03}\{ -p_{12.6} \, (p_{21.5} + p_{21.5,16}) + (1- p_{22.12} - p_{22.12.15}) \, (1- p_{11.13})\}]\}$

and

$D_2 = (-p_{24})\{\mu_0 \, [(1- p_{11.13}) \, p_{43.18} \, p_{32.7} + p_{32.7}p_{41.17}p_{13.8} + p_{12.6}\{ p_{43.18}p_{31.9} + p_{41.17}(1- p_{33.10})\} - \mu_1' \, [-p_{01} \, p_{43.18} \, p_{32.7} + p_{03} \, p_{41.17} \, p_{32.7} - + p_{02}\{ p_{43.18}p_{31.9} + (1- p_{33.10})p_{41.17}\}] + \mu_3' \, [-p_{01} \, p_{43.18} \, p_{12.6} + p_{03} \, p_{41.17} \, p_{12.6} + p_{02}\{ p_{43.18} \, (1-p_{11.13}) + p_{41.17}p_{13.8} \}] - (\mu_4' + p_{4.19} \, \mu_{19})[[p_{01} \, \{(1- p_{33.10})p_{12.6} + p_{32.7} \, p_{13.8} \} + p_{02}\{(1- p_{11.13}) \, (1- p_{33.10}) - p_{13.8}p_{31.9}\} + p_{03}\{(1- p_{11.13}) \, p_{32.7} + p_{31.9}p_{12.6} \}]\} + (1- p_{4.19}p_{19.4})\{ \mu_0 \, [(1- p_{11.13}) \, (1- p_{33.10}) \, (1- p_{22.12} - p_{22.12.15}) - (p_{23.11} + p_{23,11.14})p_{32.7}] + p_{12.6}\{ -(1- p_{33.10}) \, (p_{21.5} + p_{21.5,16}) - (p_{23.11} + p_{23,11.14})p_{31.9}\} - p_{13.8}\{ p_{32.7} \, (p_{21.5} + p_{21.5,16}) + (1- p_{22.12} - p_{22.12.15})p_{31.9}\}] + \mu_1' \, [p_{01} \, [(1- p_{33.10}) \, (1- p_{22.12} - p_{22.12.15}) - (p_{23.11} + p_{23,11.14})p_{32.7}] + p_{02}\{ (1- p_{33.10}) \, (p_{21.5} + p_{21.5,16}) + (p_{23.11} + p_{23,11.14})p_{31.9}\} + p_{03}\{ p_{32.7} \, (p_{21.5} + p_{21.5,16}) + (1- p_{22.12} - p_{22.12.15})p_{31.9}\}] + \mu_2' \, [p_{01} \, \{(1- p_{33.10}) \, p_{12.6} + p_{13.8}p_{32.7}\} + p_{02}\{(1- p_{11.13}) \, (1- p_{33.10}) - p_{13.8}p_{31.9}\} + p_{03}\{(1- p_{11.13}) \, p_{32.7} + p_{31.9}p_{12.6} \} + \mu_3' \, [p_{01} \, [p_{13.8}(1- p_{22.12} - p_{22.12.15}) + (p_{23.11} + p_{23,11.14})p_{12.6}] + p_{02}\{ (p_{21.5} + p_{21.5,16}) \, p_{13.8} + (1- p_{11.13}) \, (p_{23.11} + p_{23,11.14}) \} + p_{03}\{ -p_{12.6} \, (p_{21.5} + p_{21.5,16}) + (1- p_{22.12} - p_{22.12.15}) \, (1- p_{11.13})\}]\}$

4.2 Busy Period Analysis for Server

Let $B_i^P(t) \;\; B_i^R(t) \; B_i^S(t) \; and \;\; B_i^{HRp}(t)$ be the probabilities that the server is busy in Preventive maintenance of the system, h/w repair, s/w up-gradation and h/w replacement at an instant 't' given that the system entered state i at t = 0. The recursive relations for $B_i^P(t) \;\; B_i^R(t) \; B_i^S(t) \; and \;\; B_i^{HRp}(t)$ are as follows

$$B_i^p(t) = W_i(t) + \sum_j q_{i,j}^{(n)}(t) \copyright B_j^p(t),$$
$$B_i^R(t) = W_i(t) + \sum_j q_{i,j}^{(n)}(t) \copyright B_j^R(t)$$
(9)
$$B_i^S(t) = W_i(t) + \sum_j q_{i,j}^{(n)}(t) \copyright B_j^S(t) \;\; and$$
$$B_i^{HRp}(t) = W_i(t) + \sum_j q_{i,j}^{(n)}(t) \copyright B_j^{HRp}(t) \qquad (11)$$
(10)

Where *j* is any successive regenerative state to which the regenerative state *i* can transit through n transitions. Let $W_i(t)$ be the probability that the server is busy in state S_i due to preventive maintenance, hardware repair, software up-gradation and h/w replacement up to time t without making any transition to any other regenerative state or returning to the same via one or more non-regenerative states. We have

$$W_1 = e^{-(a\lambda_1 + b\lambda_2 + \alpha_0)t}\overline{F}(t) + (\alpha_0 e^{-(a\lambda_1 + b\lambda_2 + \alpha_0)t} \copyright 1)\overline{F}(t) +$$
$$(a\lambda_1 e^{-(a\lambda_1 + b\lambda_2 + \alpha_0)t} \copyright 1)\overline{F}(t) + (b\lambda_2 e^{-(a\lambda_1 + b\lambda_2 + \alpha_0)t} \copyright 1)\overline{F}(t)$$
$$W_2 = e^{-(a\lambda_1 + b\lambda_2 + \alpha_0 + \beta_0)t}\overline{G}(t) + (\alpha_0 e^{-(a\lambda_1 + b\lambda_2 + \alpha_0 + \beta_0)t} \copyright 1)\overline{G}(t) +$$
$$(a\lambda_1 e^{-(a\lambda_1 + b\lambda_2 + \alpha_0 + \beta_0)t} \copyright 1)\overline{G}(t) + (b\lambda_2 e^{-(a\lambda_1 + b\lambda_2 + \alpha_0 + \beta_0)t} \copyright 1)\overline{G}(t)$$
$$W_3 = e^{-(a\lambda_1 + b\lambda_2 + \alpha_0)t}\overline{H}(t) + (\alpha_0 e^{-(a\lambda_1 + b\lambda_2 + \alpha_0)t} \copyright 1)\overline{H}(t) +$$
$$(a\lambda_1 e^{-(a\lambda_1 + b\lambda_2 + \alpha_0)t} \copyright 1)\overline{H}(t) + (b\lambda_2 e^{-(a\lambda_1 + b\lambda_2 + \alpha_0)t} \copyright 1)\overline{H}(t)$$
$$W_4 = e^{-(a\lambda_1 + b\lambda_2 + \alpha_0)t}\overline{M}(t) + (\alpha_0 e^{-(a\lambda_1 + b\lambda_2 + \alpha_0)t} \copyright 1)\overline{M}(t) + (a\lambda_1 e^{-(a\lambda_1 + b\lambda_2 + \alpha_0)t} \copyright 1)\overline{M}(t)$$
$$+(b\lambda_2 e^{-(a\lambda_1 + b\lambda_2 + \alpha_0)t} \copyright 1)\overline{M}(t), \quad W_{19} = \overline{G}(t)$$

Taking LT of above relations (11) and, solving for $B_i^{*P}(s)$ $B_i^{*R}(s) \; B_i^{*S}(s) \; and \;\; B_i^{*HRp}(s)$, the time for which server is busy due to preventive maintenance, h/w repair, s/w upgradation and h/w replacement respectively is given by

$$B_0^H = \lim_{s\to 0} sB_0^{*H}(s) = \frac{N_3^H}{D_2}, \quad B_0^S = \lim_{s\to 0} sB_0^{*S}(s) = \frac{N_3^S}{D_2},$$
$$B_0^R = \lim_{s\to 0} sB_0^{*R}(s) = \frac{N_3^R}{D_2}$$
$$\text{And } B_0^{HRp} = \lim_{s\to 0} sB_0^{*HRp}(s) = \frac{N_3^{HRp}}{D_2} \qquad (12)$$

where

$N_3^P = W_1^*(0)\{p_{24}[-p_{01}p_{43.18}p_{32.7} + p_{03}p_{41.17}\,p_{32.7} + p_{02}\{p_{31.9}p_{43.18} + p_{41.17}(1-p_{33.10})\}] + (1-p_{4.19}p_{19.4})[p_{01}(1-p_{33.10})(1-p_{22.12}-p_{22.12,15}) - (p_{23.11}+p_{23,11.14})p_{32.7} + p_{02}\left\{\begin{matrix}(p_{23.11}+p_{23.11,14})p_{31.9} + (1-p_{33.10})\\(p_{21.5}+p_{21.5.15})\end{matrix}\right\} + p_{03}\{(1-p_{22.12}-p_{22.12,15})\,p_{31.9} + p_{32.7}(p_{21.5}+p_{21.5.15})\}]$

$N_3^R = W_2^*(0)(1-p_{4.19}p_{19.4})\{p_{01}[(1-p_{33.10})p_{12.6} + p_{13.8}p_{32.7}] + p_{02}[(1-p_{33.10})(1-p_{11.13}) - p_{13.8}p_{31.9}] + p_{03}[(1-p_{11.13})p_{32.7} + p_{12.6}p_{31.9})]\} + W_{19}^*(0)(p_{4.19}p_{24})\{p_{01}[(1-p_{33.10})p_{12.6} + p_{13.8}p_{32.7}] + p_{02}[(1-p_{33.10})(1-p_{11.13}) - p_{13.8}p_{31.9}] + p_{03}[(1-p_{11.13})p_{32.7} + p_{12.6}p_{31.9})]\}$

$N_3^S = W_3^*(0)[p_{24}\{p_{01}p_{43.18}p_{12.6} - p_{03}p_{41.17}\,p_{12.6} + p_{02}\{(1-p_{11.13})p_{43.18} + p_{41.17}p_{13.8}\}] + (1-p_{4.19}p_{19.4})\{p_{01}[p_{13.8}(1-p_{22.12}-p_{22.12,15}) + (p_{23.11}+p_{23,11.14})p_{12.6}] + p_{02}\left\{\begin{matrix}(1-p_{11.13})(p_{23.11}+p_{23.11,14}) + p_{13.8}\\(p_{21.5}+p_{21.5.15})\end{matrix}\right\} + p_{03}\{(1-p_{22.12}-p_{22.12,15})(1-p_{11.13}) - p_{12.6}(p_{21.5}+p_{21.5.15})\}]$

$N_3^{HRp} = W_4^* p_{24}[p_{01}\{(1-p_{33.10})p_{12.6} + p_{13.8}p_{32.7})\} + p_{02}\{(1-p_{33.10})(1-p_{11.13}) - p_{13.8}p_{31.9})\} + p_{03}\{(1-p_{11.13})p_{32.7} + p_{12.6}p_{31.9})\}]$ and D2 is already mentioned.

4.3 Expected Number of H/w replacement and s/w up-gradations

Let $R_i^H(t)$ and $R_i^S(t)$ the expected number of h/w replacements and s/w up-gradations by the server in (0, t] given that the system entered the regenerative state i at t = 0. The recursive relations for $R_i^H(t)$ and $R_i^S(t)$ are given as

$$R_i^H(t) = \sum_j Q_{i,j}^{(n)}(t) \circledR \left[\delta_j + R_j^H(t)\right],$$

$$R_i^S(t) = \sum_j Q_{i,j}^{(n)}(t) \circledR \left[\delta_j + R_j^S(t)\right] \quad 13)$$

Where j is any regenerative state to which the given regenerative state i transits and δj =1, if j is the regenerative state where the server does job afresh, otherwise δj = 0.

Taking LST of relations and, solving for $\tilde{R}_0^H(s)$ and $\tilde{R}_0^S(s)$. The expected numbers of replacements per unit time to the hardware and software failures are respectively of given by

$$R_0^H(\infty) = \lim_{s\to 0} s\tilde{R}_0^H(s) = \frac{N_4^H}{D_2} \text{ and } R_0^S(\infty) = \lim_{s\to 0} s\tilde{R}_0^S(s) = \frac{N_4^S}{D_2} \quad (14)$$

$N_4^H = (p_{40} + p_{41.17} + p_{43.18})(p_{24})\{p_{01}[(1-p_{33.10})p_{12.6} + p_{13.8}p_{32.7}] + p_{02}[(1-p_{11.13})(1-p_{33.10}) - p_{31.9}p_{13.8}] + p_{03}[(1-p_{11.13})p_{32.7} + p_{31.9}p_{12.6}]\} + (p_{22.12,15} + p_{21.5,16} + p_{23.11,14})(1-p_{19.4}p_{4.19})\{p_{01}[(1-p_{33.10})p_{12.6} + p_{13.8}p_{32.7}] + p_{02}[(1-p_{11.13})(1-p_{33.10}) - p_{31.9}p_{13.8}] + p_{03}[(1-p_{11.13})p_{32.7} + p_{31.9}p_{12.6}]\}$

$N_4^S = (p_{24})\{[p_{01}p_{43.18}p_{12.6} - p_{03}p_{41.17}\,p_{12.6} + p_{02}\left\{\begin{matrix}(1-p_{11.13})p_{43.18}\\-p_{41.17}p_{13.8}\end{matrix}\right\}] + (1-p_{19.4}p_{4.19})\{p_{01}[p_{13.8}(1-p_{22.12}-p_{22.12,15}) + (p_{23.11}+p_{23,11.14})p_{12.6}] + p_{02}\left\{\begin{matrix}(1-p_{11.13})(p_{23.11}+p_{23.11,14}) +\\p_{13.8}(p_{21.5}+p_{21.5.16})\end{matrix}\right\} p_{03}\left\{\begin{matrix}(1-p_{22.12}-p_{22.12,15})(1-p_{11.13}) - p_{12.6}\\(p_{21.5}+p_{21.5.16})\end{matrix}\right\}]$

4.4 Expected Number of H/w replacement and s/w up-gradations

Let $N_i(t)$ be the expected number of visits by the server in (0, t] given that the system entered the regenerative state i at t = 0. The recursive relations for $N_i(t)$ are given as

$$N_i(t) = \sum_j Q_{i,j}^{(n)}(t) \circledR \left[\delta_j + N_j(t)\right] \quad (15)$$

Where j is any regenerative state to which the given regenerative state i transits and δj=1, if j is the regenerative state where the server does job afresh, otherwise δj= 0. Taking LT of relation (15) and solving for $\tilde{N}_0(s)$. The expected number of visit per unit time by the server are given by

$$N_0(\infty) = \lim_{s\to 0} s\tilde{N}_0(s) = \frac{N_5}{D_2}, \text{ where} \quad (16)$$

N_5 =$(-p_{24})[(1-p_{11.13})\ p_{43.18}\ (1-p_{11.13}) + p_{32.7}p_{41.17}p_{13.8} + p_{12.6}\{p_{43.18}p_{31.9} + (1-p_{33.10})p_{41.17}\} + (1-p_{4.19}p_{19.4})\ [(1-p_{11.13})\ \{(1-p_{33.10})$

(1- $p_{22.12}$- $p_{22.12.15}$) – $p_{32.7}$ ($p_{23.11}$+ $p_{23.11,14}$)}– $p_{12.6}$ { ($p_{21.5}$+ $p_{21.5.16}$)(1- $p_{33.10}$)+ $p_{31.9}$ ($p_{23.11}$+ $p_{23.11,14}$) }- $p_{13.8}${($p_{21.5}$+ $p_{21.5.16}$) $p_{31.9}$+(1- $p_{22.12}$- $p_{22.12.15}$) $p_{32.7}$ }]

4.5 ECONOMIC ANALYSIS

The profit incurred to the system model in steady state can be obtained as

$$P = K_0A_0 - K_1B_0^P - K_2B_0^R - K_3B_0^S - K_4B_0^{HRp} - K_5R_0^H - K_6R_0^S - K_7N_0 \quad (17)$$

K_0 = Revenue per unit up-time of the system
K_1 = Cost per unit time for which server is busy due preventive maintenance
K_2 = Cost per unit time for which server is busy due to hardware failure
K_3 = Cost per unit up-gradation of the failed software
K_4 = Cost per unit replacement of the failed hardware component
K_5 = Cost per unit replacement of the failed hardware
K_6 =. Cost per unit up-gradation of the failed software
K_7 = Cost per unit visit by the server

5 CONCLUSION

In the present study, the numerical results considering a particular case $g(t)=\theta e^{-\theta t}$, $h(t)=\beta e^{-\beta t}$, $f(t)=\alpha e^{-\alpha t}$ and $m(t)=\gamma e^{-\gamma t}$ are obtained for some reliability and economic measures of a computer system of two identical units. The graphs for mean time to system failure (MTSF), availability and profit are drawn with respect to preventive maintenance rate (α) for fixed values of other parameters including a=.7 and b=.3 as shown respectively in fig. 2,3 and 4. The graphs for MTSF, availability and profit are drawn with respect to preventive maintenance rate (α) for fixed values of other parameters. These figures indicate that MTSF, availability and profit increases with the increase of preventive maintenance rate (α), maximum repair time (β_0), and h/w repair rate (θ). But the value of these measures decrease with the increase of maximum operation time (α_0) as well as interchanging the values of a and b, i.e., a=.3 and b=.7. Thus, on the basis on the results obtained for a particular case, it is suggested that the reliability and profit of a system in which chances of h/w failure are high can be improved by

(i) By adopting technique of redundancy, i.e., taking one more computer system in cold standby.
(ii) By performing preventive maintenance after a maximum operation time.
(iii) By making up-gradation of the outdated s/w by new one immediately.

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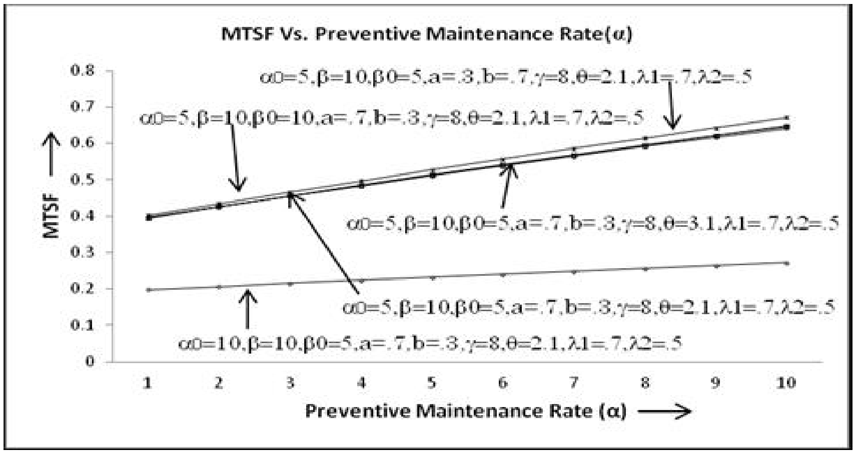


Fig. 2: MTSF vs. Preventive Maintenance Rate

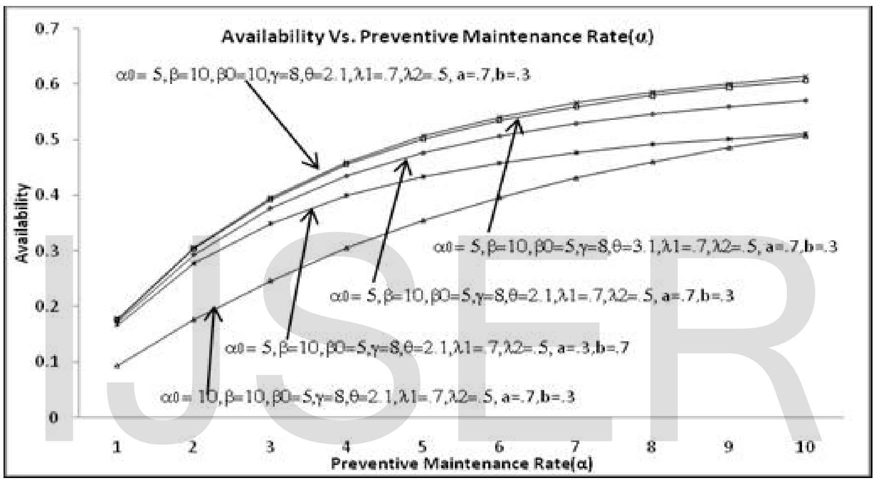


Fig. 3: Availability vs. Preventive Maintenance Rate

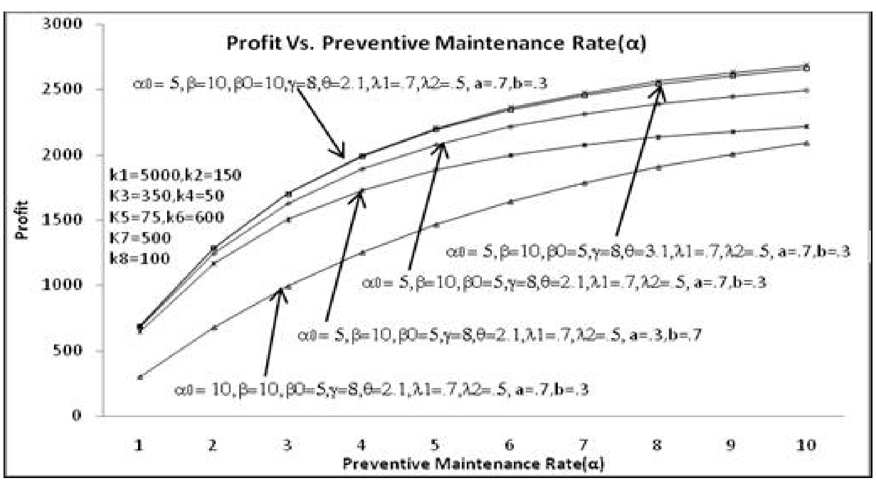
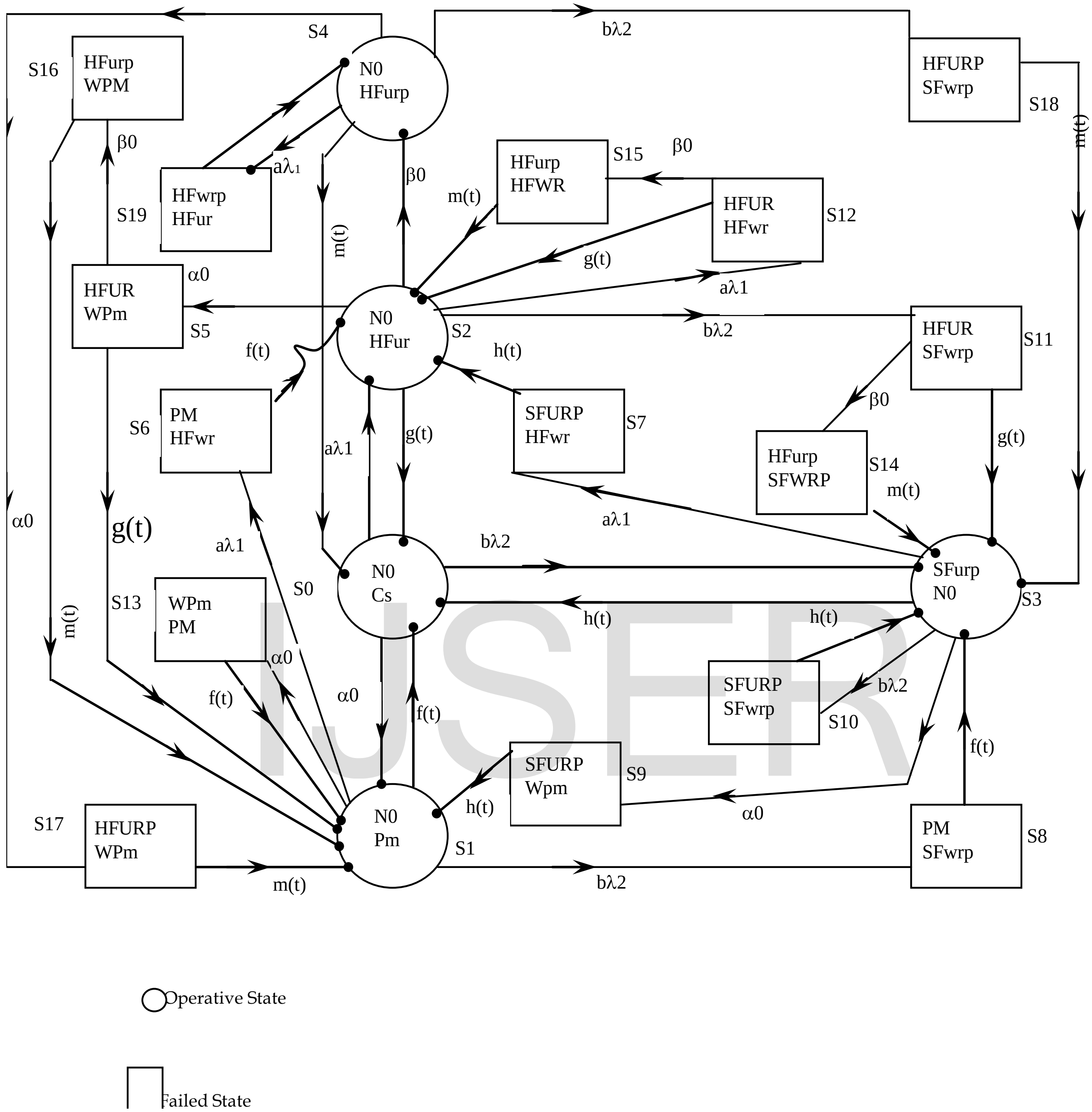


Fig. 4: Profit vs. Preventive Maintenance Rate

Figure: 1


S4
N0
HFurp
bλ2
S16
HFurp
WPM
HFURP
SFwrp
S18
m(t)
β0
aλ1
β0
HFwrp
HFur
m(t)
HFurp
HFWR
S15
β0
m(t)
HFUR
HFwr
S12
S19
g(t)
α0
aλ1
HFUR
WPm
S5
N0
HFur
S2
bλ2
HFUR
SFwrp
S11
f(t)
h(t)
β0
PM
HFwr
S6
aλ1
g(t)
SFURP
HFwr
S7
HFurp
SFWRP
S14
g(t)
m(t)
α0
g(t)
aλ1
aλ1
bλ2
N0
Cs
S0
SFurp
N0
S3
S13
WPm
PM
m(t)
α0
h(t)
h(t)
f(t)
α0
f(t)
SFURP
SFwrp
bλ2
S10
f(t)
SFURP
Wpm
S9
h(t)
α0
S17
HFURP
WPm
N0
Pm
S1
PM
SFwrp
S8
m(t)
bλ2
Operative State
Failed State